

Question Paper

Physics Multiple Correct (Maximum Marks: 32)

Question No. 1

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

A positively charged thin metal ring of radius R is fixed in the $x - y$ plane with its centre at the origin O . A negatively charged particle P is released from rest at the point $(0, 0, z_0)$ where $z_0 > 0$. Then the motion of P is : { Neglect gravity }

- A. periodic for all values of z_0 satisfying $0 < z_0 < \infty$
- B. simple harmonic for all values of z_0 satisfy $0 < z_0 \leq R$
- C. approximately simple harmonic provided $z_0 \ll R$
such that P crosses O and continues to move along the negative z -axis
- D. toward $z = -\infty$

Question No. 2

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

2 moles of He are mixed with 2 moles of H_2 in a closed adiabatic container. Initially, the mixture occupies 3 L at 27°C . The volume is suddenly decreased to $3/2$ L. Choose the correct option(s) (H_2 and He can be treated as ideal gases).

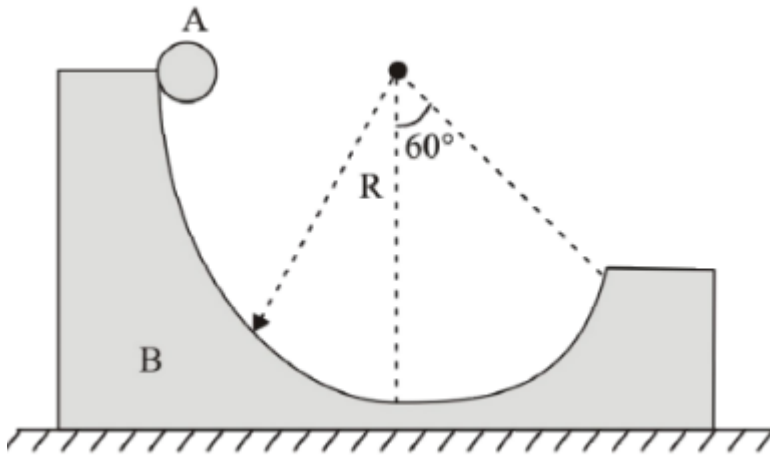
- A. γ for mixture is $3/2$
- B. final temperature = $300\sqrt{2}K$
- C. value of γ will change with higher temperature
- D. work done in compression is totally converted into internal energy.

Question No. 3

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

In the adjacent figure, a wedge 'B' of mass $2m$ is lying at rest on a horizontal surface. The wedge has a cavity which is the portion of a sphere of radius R . A small sphere 'A' of mass m is released from the top edge of the cavity to slide down. All surfaces are smooth. Point A is at the same height as centre of cavity. Choose the correct option(s).



- A. The maximum speed acquired by the sphere is $\sqrt{\frac{4gR}{3}}$
 The maximum height (from the bottom of the circular path) to which the
 B. sphere will rise after breaking off the wedge is $10R/11$
 The wedge will acquire its maximum speed when the sphere is about to
 C. leave it
 D. The maximum speed acquired by the wedge is $\sqrt{\frac{gR}{3}}$

Question No. 4

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

The density of a solid ball is to be determined in an experiment. The diameter of the ball is measured with a screw gauge, whose pitch is 0.5 mm and there are 100 division on the circular scale. The reading on the main scale is 2.3 mm and that on the circular scale is 40 divisions. If the measured mass of the ball has relative error of 1% the percentage error in density is :

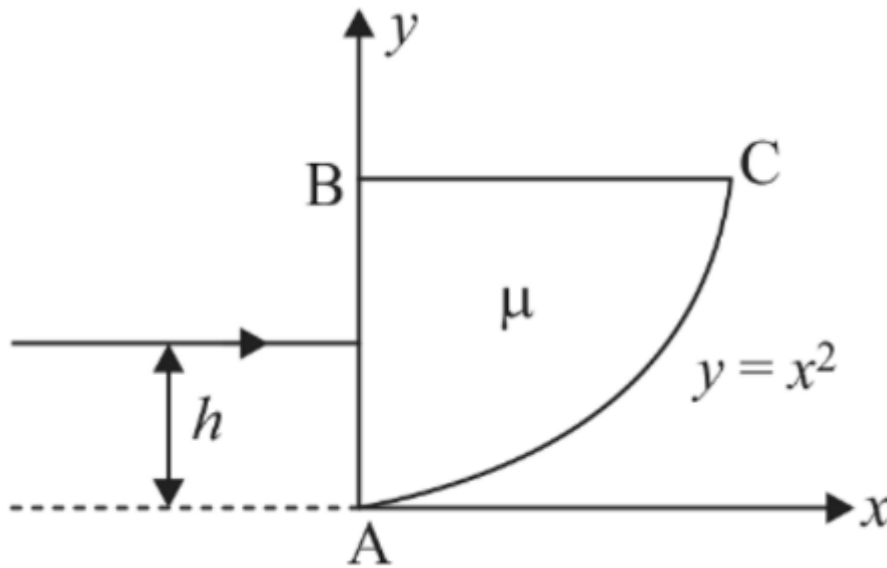
- A. 0.5%
 B. 0.8%
 C. 1.6%
 D. 2%

Question No. 5

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

The cross-section of a prism is shown in the figure. One of its refracting surface AC is given by $y = x^2$. A ray of light travelling parallel to x -axis is incident normally on the face AB . The minimum distance (h) of the incident ray from the point A to get refracted ray from surface AC is:



- A. $\frac{\mu^2 - 1}{2}$
- B. $\frac{\mu^2 - 1}{4}$
- C. $\frac{\mu^2 - 1}{8}$
- D. None of these

Question No. 6

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

A total charge q passes through a coil of resistance R . Let H be the amount of heat generated in the coil. Choose the correct statement (s).

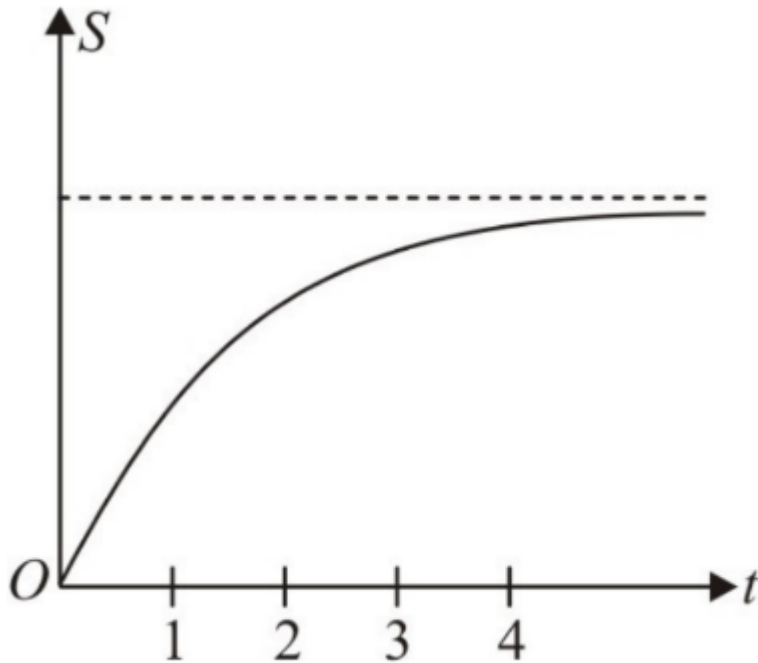
- A. $H = \frac{2q^2 R}{3t_0}$ if current decreases uniformly to zero during a time interval t_0
- B. $H = \frac{4q^2 R}{3t_0}$ if current decreases uniformly to zero during a time interval t_0
- $H = \frac{q^2 R}{t_0}$ if the current decreases smoothly down to zero halving its values
- C. every t_0 seconds
- $H = \frac{q^2 R \ln(2)}{2t_0}$ if the current decreases smoothly down to zero halving its
- D. value every t_0 seconds

Question No. 7

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

The displacement of a particle as a function of time is shown in the figure. It indicates :



The particle starts with a certain velocity, but the motion is retarded and

- A. finally the particle stops
- B. The velocity of the particle decreases
- C. The acceleration of the particle is in opposite direction to the velocity

The particle moves with a constant velocity, the motion is accelerated and

- D. finally the particle moves with another constant velocity

Question No. 8

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

An engine whistling at a constant frequency n_0 and moving with a constant velocity goes past a stationary observer. As the engine crosses him, the frequency of the sound heard by him changes by a factor $f(= \frac{n_2}{n_1})$. The actual difference in the frequencies of the sound heard by him before and after the engine crosses him is : [n_1 and n_2 are the frequencies of whistle before and after engine crosses the observer, respectively]

- A. $\frac{1}{2f} n_0(1 + f^2)$
- B. $\frac{1}{2f} n_0(1 - f^2)$
- C. $n_0(\frac{1-f}{1+f})$
- D. $\frac{1}{2} n_0(\frac{1-f}{1+f})$

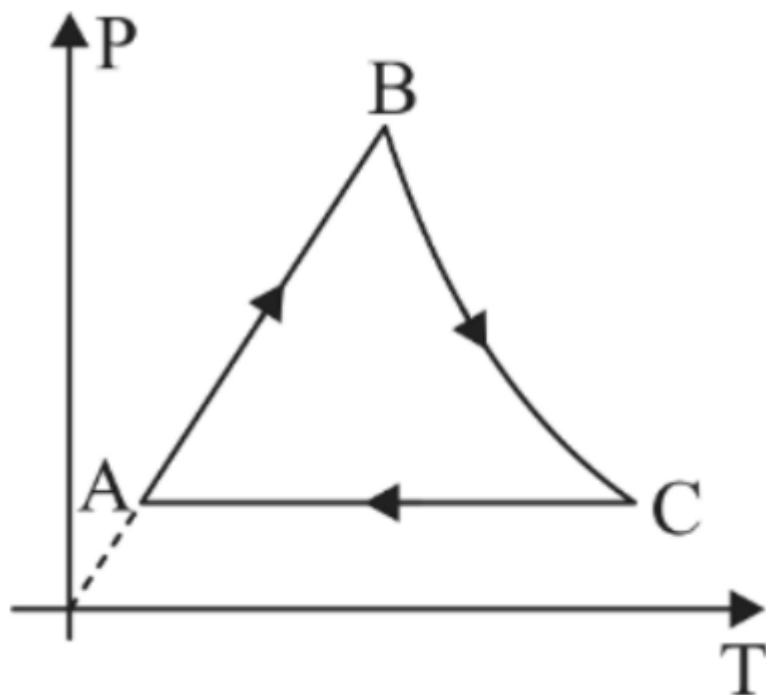
Physics Numerical (Maximum Marks: 24)

Question No. 1

Numerical Type

The answer has to be filled into the input box provided below.

A monoatomic gas initially in state $A(P_0, V_0)$ is taken through a cyclic process $ABCA$ as shown in the figure. The pressure of point B is twice of that of A . The curve BC is a rectangular hyperbola. If the net work done by the gas is $xP_0 V_0$, then find the value of x .



Question No. 2

Numerical Type

The answer has to be filled into the input box provided below.

A long coaxial cable consists of two thin walled, conducting cylinders with inner radius 2 cm and outer radius 8 cm. The inner cylinder carries a steady current 1 A, and the outer cylinder provides the return path for that current. The current produces a magnetic field between the two cylinders. The energy stored in the magnetic field for length 1 m of the cable is $20x$ nJ. The value of x is _____. (Use $\ln 2 = 0.7$)

Question No. 3

Numerical Type

The answer has to be filled into the input box provided below.

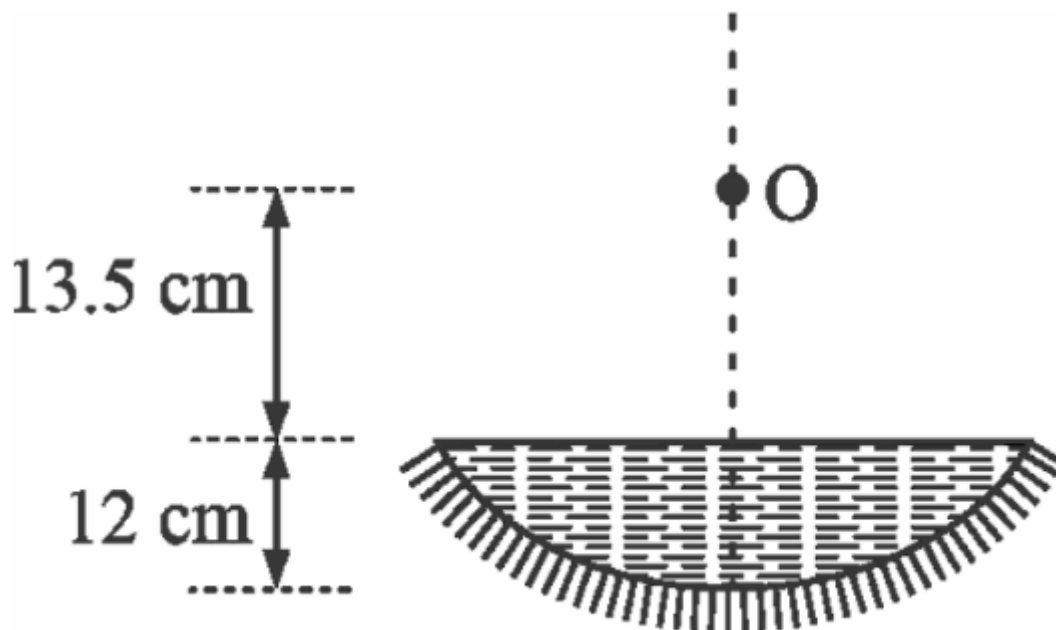
A container filled with air under pressure P_0 contains a soap bubble of radius R . The air pressure has been reduced to half isothermally and the new radius of the bubble becomes $\frac{5R}{4}$. If the surface tension of the soap water solution is S , P_0 is found to be $\frac{24nS}{R}$ SI unit. Find n

Question No. 4

Numerical Type

The answer has to be filled into the input box provided below.

A concave mirror of radius of curvature 40 cm is filled with water ($n = \frac{4}{3}$) upto a height of 12 cm. A point object O is kept on the principal axis of the mirror at height 13.5 cm from the water surface the final image formed after all reflection and refractions is situated at $6x$ cm above the water surface. Value of x _____.



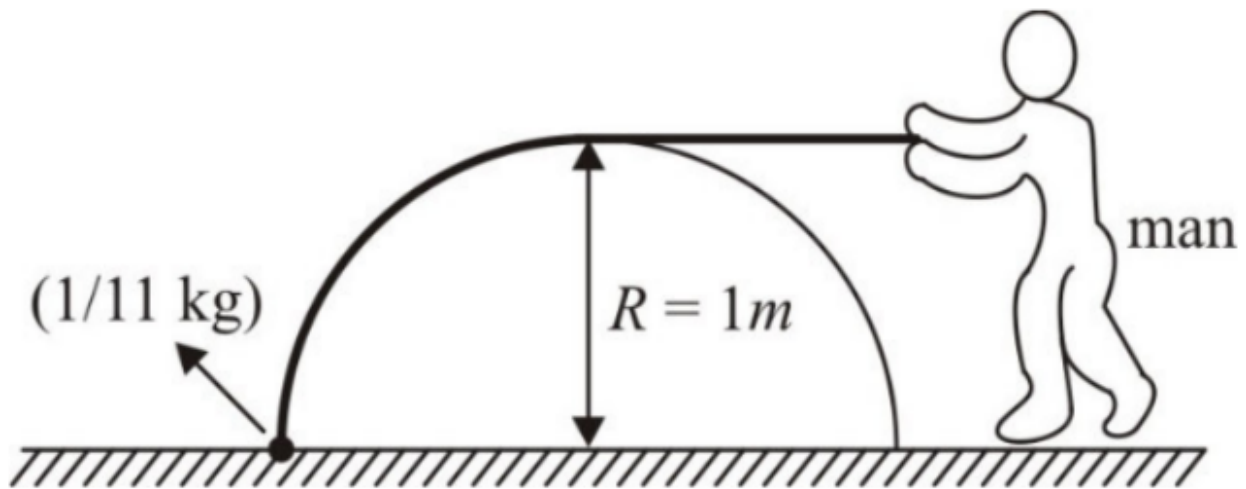
Question No. 5

Numerical Type

The answer has to be filled into the input box provided below.

A person is pulling a mass $m = \frac{1}{11}$ kg from ground as shown in the figure on a fixed rough hemispherical surface upto to the top of the hemisphere with the help of light inextensible string. Assuming block is pulled with negligible velocity, find the work done (in joule) by tension in the string.

(Given that $\mu = 0.1$)



Question No. 6

Numerical Type

The answer has to be filled into the input box provided below.

Two rigid walls at a distance of 1 m act as a heat reservoir at 100°C and 0°C . A well lagged uniform rod having length 1 m at 0°C is held between both wall. It is found that when the system reaches steady state, the supporting force can be removed and limiting friction acts on the rod at both ends. (Given : $\alpha = 10^{-5}/^\circ\text{C}$, $Y = 2 \times 10^{11}\text{ N/m}^2$, $\mu_{\text{walls}} = 0.1$ and Area of cross section $= 4\text{ mm}^2$). What is the mass of rod (in kg) ?



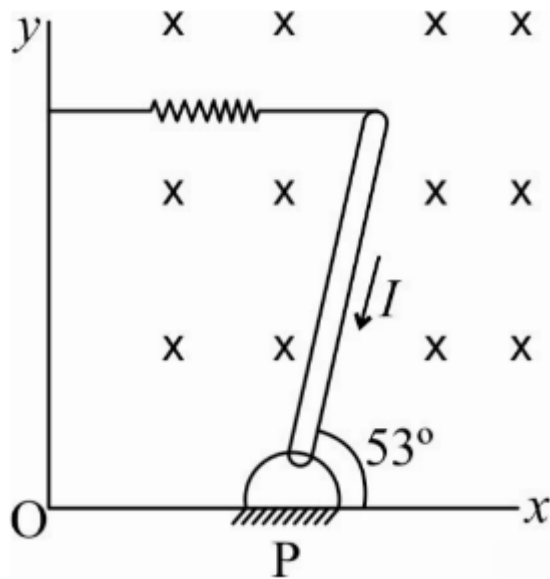
Physics Paragraph Type (Maximum Marks: 12)

Question No. 1

Only One Option Correct Type

Each question has multiple options out of which ONLY ONE is correct.

A thin, uniform rod of mass 3 kg and length 2 m is attached to hinged at point P and is kept on horizontal smooth floor. A spring of constant $k = 8 \text{ N/m}$ is connected to another end of rod with another fixed wall. The rod is in uniform magnetic field $B = 0.680 \text{ T}$ directed into plane of paper. There is current $I = 13 \text{ A}$ in rod. The figure shown represents equilibrium position of rod.



When rod is in equilibrium, which of the following is not correct?

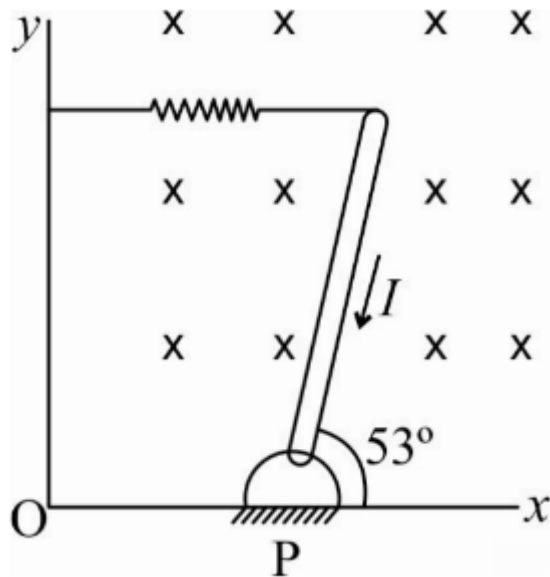
- A. Torque due to magnetic force on rod about P is $17.68 \text{ N} - \text{m}$
- B. When rod is in equilibrium, spring is stretched by 1.38 m
- C. Energy stored in spring at equilibrium is 16 J
- D. Net force on rod is zero at equilibrium

Question No. 2

Only One Option Correct Type

Each question has multiple options out of which ONLY ONE is correct.

A thin, uniform rod of mass 3 kg and length 2 m is attached to hinged at point P and is kept on horizontal smooth floor. A spring of constant $k = 8 \text{ N/m}$ is connected to another end of rod with another fixed wall. The rod is in uniform magnetic field $B = 0.680 \text{ T}$ directed into plane of paper. There is current $I = 13 \text{ A}$ in rod. The figure shown represents equilibrium position of rod.



If current in rod is increased slightly assuming spring remains horizontal :

- A. magnetic force on rod increases
- B. Torque on rod due to magnetic force increase
- C. All the above
- D. None of the above

Question No. 3

Only One Option Correct Type

Each question has multiple options out of which ONLY ONE is correct.

A solid sphere of mass M and radius R is initially at rest. Solid sphere is gradually lowered onto a truck moving with constant velocity v_0 .

(Given : Length of the plank of truck is long enough for pure rolling).



What is the final speed of the sphere's centre of mass in ground frame when eventually pure rolling starts :

- A. $\frac{5}{7} v_0$
- B. $\frac{2}{7} v_0$
- C. $\frac{7}{5} v_0$
- D. $\frac{7}{2} v_0$

Question No. 4

Only One Option Correct Type

Each question has multiple options out of which ONLY ONE is correct.

A solid sphere of mass M and radius R is initially at rest. Solid sphere is gradually lowered onto a truck moving with constant velocity v_0 .

(Given : Length of the plank of truck is long enough for pure rolling).



If a sphere with twice the radius and four times the mass had been used; what would have been its final speed?

- A. $\frac{5}{7} v_0$
- B. $\frac{2}{7} v_0$
- C. $\frac{5}{7} v_0$
- D. $\frac{7}{2} v_0$

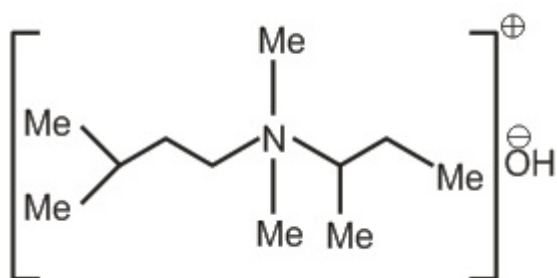
Chemistry Multiple Correct (Maximum Marks: 32)

Question No. 1

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

Give the major alkene, resulting from the thermal decomposition of hydroxide salt of the following.



- A.
- B.
- C.
- D.

Question No. 2

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

A solution containing Cu^{2+} and $\text{C}_2\text{O}_4^{2-}$ ions consumed 20 mL of $\frac{M}{4}$ KMnO_4 solution in acidic medium. The resulting solution is treated with excess of KI after neutralization. The evolved I_2 is then absorbed in 25 mL of M/10 hypo solution. Which of the following statements are correct?

- The difference of the number of milli mole of Cu^{2+} and $\text{C}_2\text{O}_4^{2-}$ ions in the
- A. solution is 10 milli mole

The difference of the number of milli mole of Cu^{2+} and $\text{C}_2\text{O}_4^{2-}$ ions in the B. solution is 25.5 milli mol.

The equivalent weight of Cu^{2+} ions in the titration with KI is equal to the C. atomic weight of Cu^{2+}

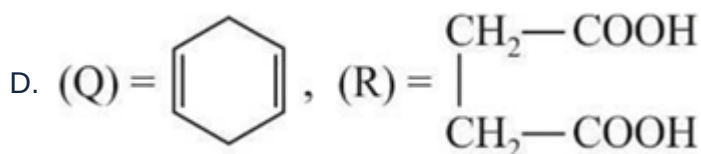
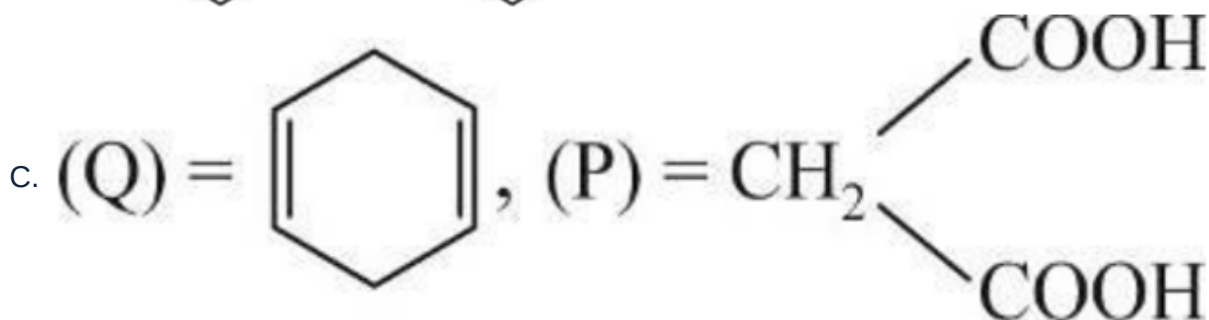
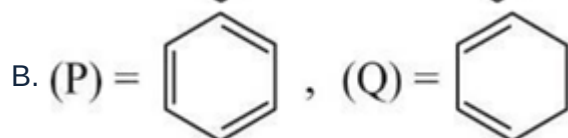
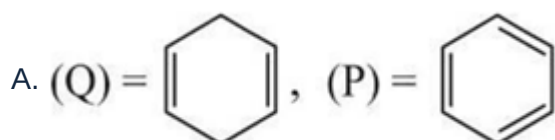
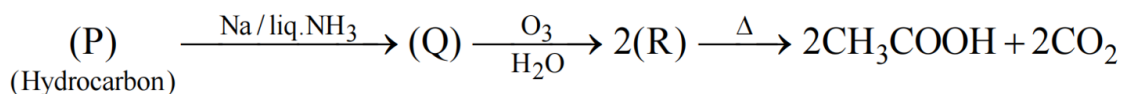
The equivalent weight of KI in the titration is $\frac{M}{2}$ (M = Molecular weight D. of KI)

Question No. 3

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

Identify the unknowns in the following reaction sequence.



Question No. 4

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

Which one is(are) correct?

A. $2.303 \log K = -\Delta H^\circ/RT + \Delta S^\circ/R$

B. $\Delta G^\circ = -2.303RT \log K$

C. $-2.303 \log K = -\Delta H^\circ/RT^2 + \Delta S^\circ/R$

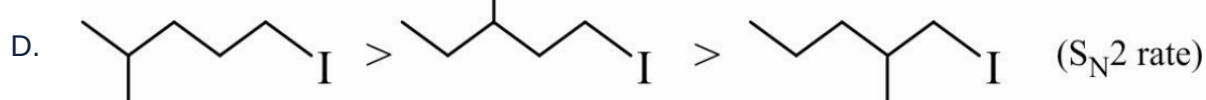
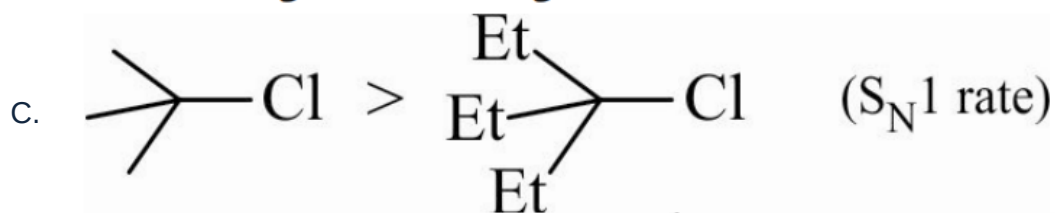
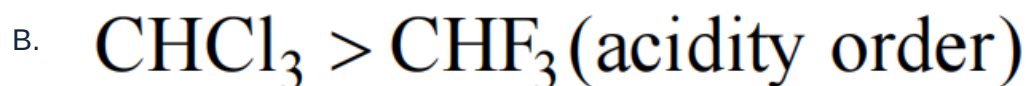
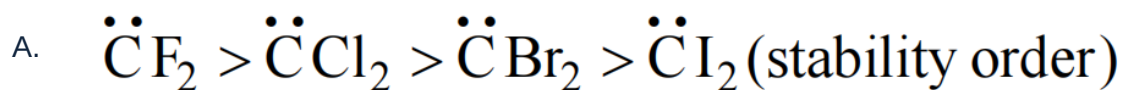
D. $2.303 \log K = (1/RT)(\Delta H^\circ + \Delta S^\circ)$

Question No. 5

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

Which of the following is/are correct statement (s) ?

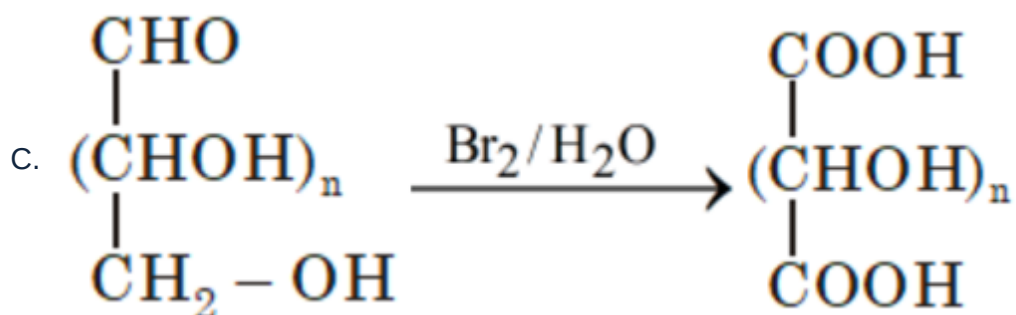
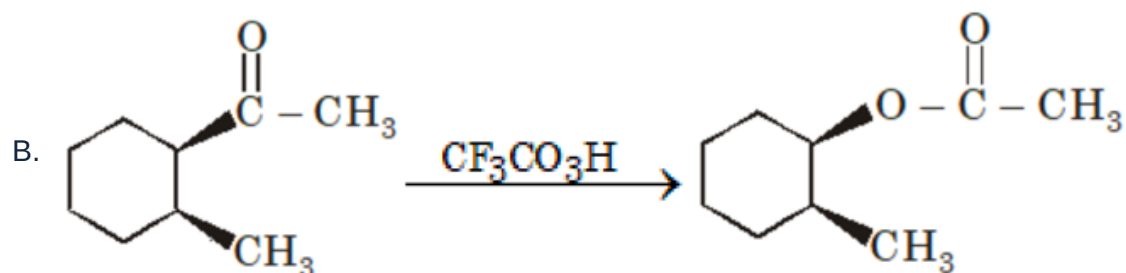
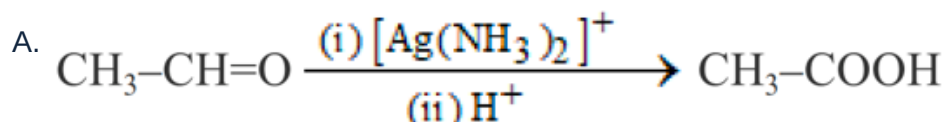


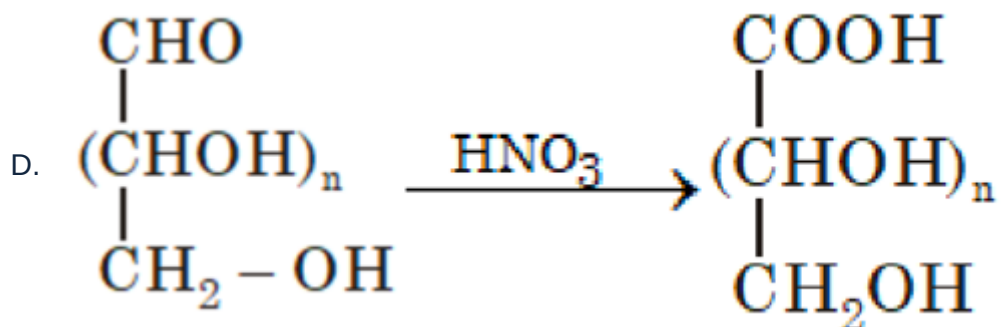
Question No. 6

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

In which of the following reaction(s) the product is/are correctly matched?





Question No. 7

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

Select the correct statement (s):

- Radial function $[R(r)]$ a part of wave function is dependent on quantum
 - A. number n only
 - Angular function depends only on the direction and is independent to the
 - B. distance from the nucleus
 - $\psi^2(r, \theta, \phi)$ is the probability density of finding the electron at a particular
 - C. point in space
 - Radial distribution function $(4\pi r^2 R^2)$ gives the probability of the electron
 - D. being present at a distance r from the nucleus

Question No. 8

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

The following reaction is a slow reaction and hence Ruthenium (molar mass = 101 gm) is used as catalyst $\text{NaBH}_4(\text{aq}) + 2\text{H}_2\text{O}(\ell) \xrightarrow{\text{Catalyst}} \text{Na}^+(\text{aq}) + \text{BO}_2^-(\text{aq}) + 4\text{H}_2(\text{g})$

Kinetic studies show that reaction is 1st order with respect to catalyst but zero-order with respect to substrate. The rate of hydrogen production per mole of Ruthenium is $92 \text{ mol H}_2(\text{molRu})^{-1} \text{ min}^{-1}$ at 27°C . Identify CORRECT option(s) $\{R = 0.08 \text{ atm} - \ell/\text{mol} - \text{K}\}$

- Approximately 0.045 mgRu, must be added to
 - 0.1 dm^3 , 1 mol dm^{-3} NaBH_4 to supply H_2 gas at rate of $0.1 \text{ dm}^3 \text{ min}^{-1}$
 - A. at 27°C and 1 atm
 - Approximately 4.574 mg of Ru, must be added to
 - 0.1 dm^3 , 1 mol dm^{-3} NaBH_4 to supply H_2 gas at rate of $0.1 \text{ dm}^3 \text{ min}^{-1}$
 - B. at 27°C and 1 atm

If 0.1 dm^3 , 1 mol dm^{-3} NaBH_4 is used to supply H_2 at a rate of $0.1 \text{ dm}^3 \text{ min}^{-1}$ at 27°C , 1 atm , then upto 96 min hydrogen can be supplied in this way

If 0.1 dm^3 , 1 mol dm^{-3} NaBH_4 is used to supply H_2 at a rate of $0.1 \text{ dm}^3 \text{ min}^{-1}$ at 27°C , 1 atm , then upto 48 min hydrogen can be supplied in this way

Chemistry Numerical (Maximum Marks: 24)

Question No. 1

Numerical Type

The answer has to be filled into the input box provided below.

0.01 mole of FeS_n (iron (II) sulphide) required 0.06 mole of AO_4^{3-} for complete oxidation. The species formed are FeO , SO_2 and A^{2+} . Calculate the present of sulphur in FeS_n .

[Atomic mass : Fe = 56, S = 32]

Question No. 2

Numerical Type

The answer has to be filled into the input box provided below.

50ml of solution of 0.2 M NaOH is mixed with 50ml of $0.1 \text{ M La}(\text{NO}_3)_3$ (aq.) solution. How many millimoles of La^{3+} ions will be precipitated.

[K_{sp} of $\text{La}(\text{OH})_3 = 10^{-16}$]

Question No. 3

Numerical Type

The answer has to be filled into the input box provided below.

When borax is heated in a platinum loop, the transparent bead is formed which is coloured. How many metal salts given below shall give a coloured bead?

NiCl_2 , CuSO_4 , KCl , $\text{Mg}(\text{NO}_3)_2$, FeSO_4 , CoSO_4 , BeCl_2 , ZnO

Question No. 4

Numerical Type

The answer has to be filled into the input box provided below.

Two isomeric compounds having vapour density equal to 45 gave the following results on analysis, C = 40%, H = 6.67% and rest is oxygen. Each of them gives effervescence with NaHCO_3 and gives monoester derivative. Compound (A) on heating with NaOH/I_2 produced iodoform and disodium oxalate while (B) on heating produced an unsaturated acid which on decarboxylation gives ethylene. If (A) and (B) both are heated separately then the total number of stereoisomers obtained from (A) and (B) are

Question No. 5

Numerical Type

The answer has to be filled into the input box provided below.

Polydentate ligands almost always form ring like structures with the central metal ion. If crowding of ligands is not considered then more the number of rings, more will be stability of complex. The hexadentate (edta)⁴⁻ ligand complexes with the Fe^{3+} ion to give stable $[\text{Fe}(\text{edta})]^-$. The number of rings formed in such a complex would be

Question No. 6

Numerical Type

The answer has to be filled into the input box provided below.

A sulphate of a metal (M) on heating evolves two gases (B) and (C) and an oxide (D). Gas (B) turns acidified dichromate paper green while gas (C) forms a trimer. Compound (D) forms a Lewis acid (E) with HCl(g) .

If, the number of $\text{S} - \text{O} - \text{S}$ bonds in trimer of gas (C) be ' n ' and the number of equivalent $\text{M} - \text{Cl}$ bonds in one molecule of anhydrous (E) be ' m ' Then, $(m + n)$ is

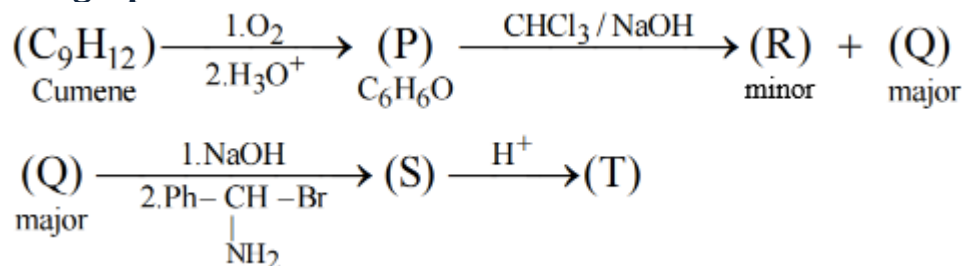
Chemistry Paragraph Type (Maximum Marks: 12)

Question No. 1

Only One Option Correct Type

Each question has multiple options out of which ONLY ONE is correct.

Paragraph:



Question:

Which of the following statement is correct about the given reaction sequence?

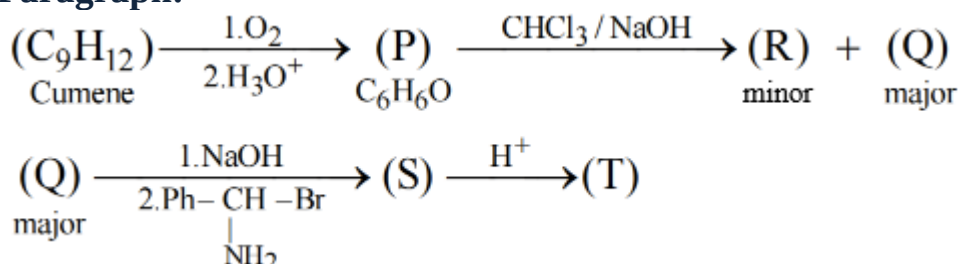
- A. (P) is a steam volatile compound.
- B. (Q) & (R) are stereoisomers.
- C. (Q) gives dark violet colouration with 1% aqueous FeCl_3 solution.
- D. (R) & (S) can be distinguished by Tollen's reagent.

Question No. 2

Only One Option Correct Type

Each question has multiple options out of which ONLY ONE is correct.

Paragraph:



Question:

Which of the following is not true about the product(s) in the above reaction sequence?

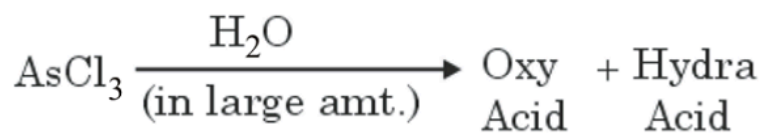
- A. Formation of (P) from cumene involves rearrangement.
- B. (Q) & (S) can be distinguished by 1% aqueous FeCl_3 solution.
- C. Formation of (S) from (Q) involves nucleophilic substitution.
- D. (T) has degree of unsaturation equal to 9 .

Question No. 3

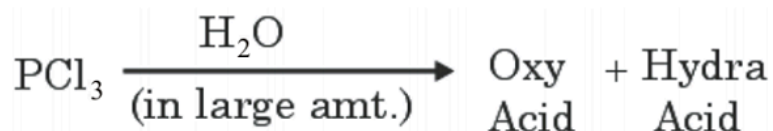
Only One Option Correct Type

Each question has multiple options out of which ONLY ONE is correct.

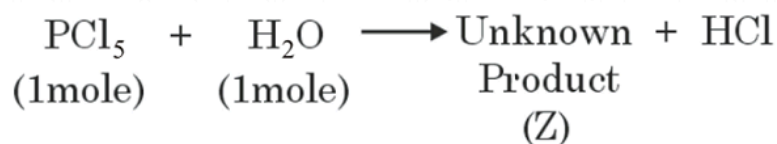
Paragraph:



↓
Basicity(X)



↓
Basicity(Y)



Question:

The value of $\left(\frac{X}{Y}\right)$ is.

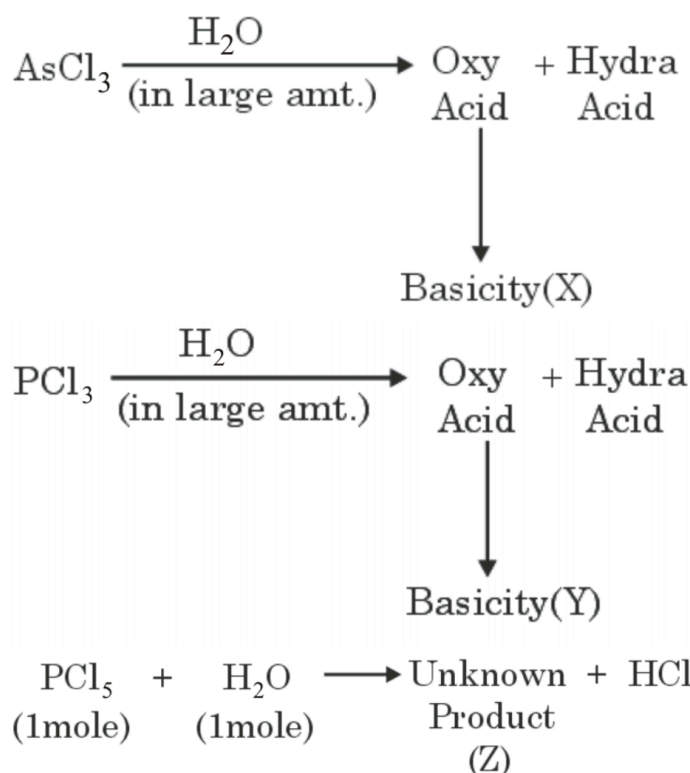
- A. 1.5
- B. 2
- C. 0.5
- D. 0.25

Question No. 4

Only One Option Correct Type

Each question has multiple options out of which ONLY ONE is correct.

Paragraph:



Question:

The ratio of number of π bond(s) to number of σ bond(s) in ' Z ' is

- A. 1.5
- B. 2
- C. 0.5
- D. 0.25

Mathematics Multiple Correct (Maximum Marks: 32)

Question No. 1

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

Let $P = \sum_{n=1}^{10} n \left\{ \frac{1^2}{1+n} + \frac{2^2}{2+n} + \dots + \frac{10^2}{10+n} \right\}$, then which of the following is/are correct ?

- A. $2P$ is perfect square
- B. $2P$ is divisible by 11^2
- C. Number of divisors of $2P$ is 9
- D. The remainder when $2P$ is divided by 6 is 1

Question No. 2

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

Consider points $A(0, 0, 0)$, $B(1, 1, 1)$ and plane $P : x - y + z = 3$. Let $Q(a, b, c)$ be a moving point on plane P then identify the correct statement(s)

- A. If $QA + QB$ is minimum then $a - c - 10b$ is equal to 1
- B. If $QA + QB$ is minimum then $a - c - 10b$ is equal to 2
- C. If $|QA - QB|$ is maximum then $a - 2b + c$ is equal to 0
- D. If $|QA - QB|$ is minimum then Q is unique

Question No. 3

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

A hyperbola intersects an ellipse $x^2 + 9y^2 = 9$ orthogonally. The eccentricity of the hyperbola is reciprocal of that of ellipse. If the axes of the hyperbola are along coordinate axes, then

- A. vertices of hyperbola are $(\pm \frac{8}{3}, 0)$
- B. y coordinate of point of intersection of ellipse and hyperbola is either $\frac{1}{3}$ or $-\frac{1}{3}$
- C. latus rectum of hyperbola is $\frac{2}{3}$
- D. latus rectum of hyperbola is $\frac{4}{3}$

Question No. 4

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

If the series of natural numbers is divided into groups $(1), (2, 3, 4, 5, 6, 7, 8, 9), (10, 11, 12, 13, 14, \dots, 36), \dots$ and so on, then -

- A. total number of natural numbers in first n groups is $(\frac{n(n+1)}{2})^2$
- B. first term in n^{th} group is $\frac{1}{4}(n-1)^2n^2 + 1$
- C. last term in n^{th} group is $\frac{1}{4}(n-1)^2n^2 + n^3$
- D. the sum of the numbers in n^{th} group is $\frac{n^3}{4}(n^4 + n^2 + 2)$

Question No. 5

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

In $\triangle ABC$, if

$$\angle B = \sec^{-1}\left(\frac{5}{4}\right) + \operatorname{cosec}^{-1} \sqrt{5}, \angle C = \operatorname{cosec}^{-1}\left(\frac{25}{7}\right) + \cot^{-1}\left(\frac{9}{13}\right) \quad \text{and} \\ c = 3.$$

(All symbols used have their usual meaning in a triangle.) which of the following is/are correct

A. $\tan A, \tan B, \tan C$ are in A.P.

B. $\tan A, \tan B, \tan C$ are in G.P.

The distance between orthocentre and centroid of triangle with sides $a^2, b^{\frac{4}{3}}$
C. and c is equal to $\frac{5}{3}$

The distance between orthocentre and centroid of triangle with sides $a^2, b^{\frac{4}{3}}$
D. and c is equal to $\frac{10}{3}$

Question No. 6

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

The complex numbers z_1, z_2, \dots, z_n , represents the vertices of a regular polygon of n sides in order, inscribed in a circle of unit radius and

$z_3 + z_n = Az_1 + Az_2$, which of the following statement/s is/are correct

A. if $n = 4$ then $A = \sqrt{5}$

B. if $n = 6$ then $A = \sqrt{4}$

C. if $n = 8$ then $A = \sqrt{3}$

D. if $n = 12$ then $A = \sqrt{2}$

Question No. 7

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

A bag contains 100 articles of which 90 are good and 10 are defective. The articles are tested one by one till all defective are obtained. Which of the following options are correct?

A. Probability that first defective article is obtained in second testing is $\frac{1}{11}$

The probability that a particular defective article is obtained in 87th testing
B. is $\frac{1}{100}$

Given that second article is defective the probability that first article was
C. good is $\frac{10}{11}$

Given that second article is defective the probability that first article was
D. defective is $\frac{1}{11}$

Question No. 8

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

Let $f(x) : \mathbb{R} \rightarrow [-1, 1]$ be twice differentiable and $f^2(0) + (f'(0))^2 = 4$, then which of the following(s) is/are TRUE?

- There exist $\alpha, \beta \in \mathbb{R}$ where $\alpha < \beta$, such that f is one-one on the interval
- A. (α, β)
 - B. There exists $c \in (0, 2)$ such that $|f'(c)| \leq 1$
 - C. $\lim_{x \rightarrow \infty} f(x) = 1$
 - D. There exists some x_0 such that $f(x_0) + f''(x_0) = 0$ but $f'(x_0) \neq 0$

Mathematics Numerical (Maximum Marks: 24)

Question No. 1

Numerical Type

The answer has to be filled into the input box provided below.

If A and B are two matrices of order 3 such that $2A + 3BB^T = I$ and $B^{-1} = A^T$.

Let $\alpha = \det(A^{-1} - 3B^3 + BA)$ and $\beta = \text{Tr}(A^{-1} - 3B^3 - BA)$, then value of $\frac{\alpha}{\beta}$ is

Question No. 2

Numerical Type

The answer has to be filled into the input box provided below.

If $\int 4(3 - 2x)^{-2} \left(\frac{3-2x}{3+2x}\right)^{\frac{1}{3}} dx = \frac{3}{\alpha} \left(\frac{3+2x}{3-2x}\right)^{\frac{\beta}{\gamma}} + c$ (where c is constant of integration and β, γ are prime numbers), then value of $(\alpha + \gamma - \beta)$ is

Question No. 3

Numerical Type

The answer has to be filled into the input box provided below.

Consider the statistics of two sets of observations as follows:

	Size	Mean	Variance
Observation I	10	2	2
Observation II	n	3	1

If the variance of the combined set of these two observations is $\frac{17}{9}$, then the value of n is equal to ____.

Question No. 4

Numerical Type

The answer has to be filled into the input box provided below.

The absolute value of the expression $y^3 \frac{d^2y}{dx^2}$ for the ellipse $3x^2 + 4y^2 = 12$ is $\frac{P}{Q}$ (where P and Q are coprime) then the value of $(P - Q)$ is

Question No. 5

Numerical Type

The answer has to be filled into the input box provided below.

If $x = (2 + \sqrt{3})^n$, $n \in \mathbb{N}$ then the value of $x - x^2 + x[x]$, (Where $[.]$ denotes greatest integer function) is equal to

Question No. 6

Numerical Type

The answer has to be filled into the input box provided below.

If L.C.M. of $(a, b) = 5^2 \times 3^4$, L.C.M. of $(b, c) = 5^3 \times 3^4$ & L.C.M. of $(c, a) = 5^3 \times 3^4$, then possible ordered triplets of (a, b, c) are k then $\frac{k}{5}$ is equal to

Mathematics Paragraph Type (Maximum Marks: 12)

Question No. 1

Only One Option Correct Type

Each question has multiple options out of which ONLY ONE is correct.

Paragraph:

Let $f(x) = \frac{(x-a)(x-b)}{(x-c)(x-d)}$ where $a < b < c < d$. Column-I: contains information about real roots of the equation; Column-II: contains information about limit of $f(x)$; Column-III: contains information about increasing interval and decreasing interval of $f(x)$ and $f'(x)$ and information about local minimum value and local maximum value of $f(x)$

Column-I		Column-II		Column-III	
(I)	$f(x) = 1$ has exactly one real root	(i)	$\lim_{x \rightarrow c^-} f(x) = -\infty$	(P)	$f(x)$ is increasing in the interval (b, c)
(II)	$f(x) = 2$ has exactly two real root	(ii)	$\lim_{x \rightarrow d^+} f(x) = \infty$	(Q)	$f(x)$ is decreasing in the interval (d, ∞)
(III)	$f(x) = -10$ has one real root	(iii)	$\lim_{x \rightarrow d^-} f(x) = -\infty$	(R)	$f'(x)$ is increasing in the interval (b, c)
(IV)	$f(x) = 10$ has two real root	(iv)	$\lim_{x \rightarrow \infty} f(x) = 1$	(S)	local minimum value of $f(x)$ is smaller than local maximum value of $f(x)$

Question:

Which of the following options is the only CORRECT combination?

- A. (I) (ii) (S)
- B. (II) (iii) (Q)
- C. (I) (i) (R)
- D. (III) (iv) (S)

Question No. 2

Only One Option Correct Type

Each question has multiple options out of which ONLY ONE is correct.

Paragraph:

Let $f(x) = \frac{(x-a)(x-b)}{(x-c)(x-d)}$ where $a < b < c < d$. Column-I: contains information about real roots of the equation; Column-II: contains information about limit of $f(x)$; Column-III: contains information about increasing interval and decreasing interval of $f(x)$ and $f'(x)$ and information about local minimum value and local maximum value of $f(x)$

Column-I		Column-II		Column-III	
(I)	$f(x) = 1$ has exactly one real root	(i)	$\lim_{x \rightarrow c^-} f(x) = -\infty$	(P)	$f(x)$ is increasing in the interval (b,c)
(II)	$f(x) = 2$ has exactly two real root	(ii)	$\lim_{x \rightarrow d^+} f(x) = \infty$	(Q)	$f(x)$ is decreasing in the interval (d,∞)
(III)	$f(x) = -10$ has one real root	(iii)	$\lim_{x \rightarrow d^-} f(x) = -\infty$	(R)	$f'(x)$ is increasing in the interval (b,c)
(IV)	$f(x) = 10$ has two real root	(iv)	$\lim_{x \rightarrow \infty} f(x) = 1$	(S)	local minimum value of $f(x)$ is smaller than local maximum value of $f(x)$

Question:

Which of the following options is the only INCORRECT combination?

- A. (II) (ii) (P)
- B. (III) (i) (R)
- C. (IV) (iv) (P)
- D. (I) (iii) (Q)

Question No. 3

Only One Option Correct Type

Each question has multiple options out of which ONLY ONE is correct.

Paragraph:

Let $f(x)$ be a double differentiable function with continuous second derivative defined on the interval $[0, 1]$ satisfying the equation

$$\int_0^x \sqrt{1 - (f''(t))^2} dt = \int_0^x f'(t) dt \quad \forall x \in [0, 1] \quad \text{and} \quad f(0) = f'(0) = 0.$$

Column-I: contains the information about interval of value of $f(x)$; Column-II: contains information about value of $f(x)$; Column-III: contains information about interval of value of definite integral of $f(x)$; Match the following Column(s)

Column-I		Column-II		Column-III
(I)	$f\left(\frac{1}{2}\right) < \frac{1}{8}$	(i)	$f\left(\frac{1}{3}\right) = \frac{1}{18}$	(P) $\int_0^1 f(x) dx > 1$
(II)	$f\left(\frac{1}{2}\right) > \frac{1}{8}$	(ii)	$f\left(\frac{1}{4}\right) = \frac{1}{8}$	(Q) $\int_0^1 f(x) dx > \frac{1}{6}$
(III)	$f\left(\frac{1}{3}\right) < \frac{1}{18}$	(iii)	$f\left(\frac{1}{6}\right) = 2\sin^2\left(\frac{1}{12}\right)$	(R) $\int_0^1 f(x) dx < \frac{1}{6}$
(IV)	$f\left(\frac{1}{4}\right) < \frac{1}{4}$	(iv)	$f\left(\frac{1}{8}\right) = 2\sin^2\left(\frac{1}{16}\right)$	(S) $\int_0^1 f(x) dx < \frac{1}{4}$

Question:

Which of the following options is the only CORRECT combination?

- A. (I) (ii) (Q)
- B. (II) (i) (S)
- C. (IV) (ii) (Q)
- D. (III) (iii) (R)

Question No. 4

Only One Option Correct Type

Each question has multiple options out of which ONLY ONE is correct.

Paragraph:

Let $f(x)$ be a double differentiable function with continuous second derivative defined on the interval $[0, 1]$ satisfying the equation

$$\int_0^x \sqrt{1 - (f''(t))^2} dt = \int_0^x f'(t) dt \quad \forall x \in [0, 1] \quad \text{and} \quad f(0) = f'(0) = 0.$$

Column-I: contains the information about interval of value of $f(x)$; Column-II: contains information about value of $f(x)$; Column-III: contains information about interval of value of definite integral of $f(x)$; Match the following Column(s)

Column-I		Column-II		Column-III
(I)	$f\left(\frac{1}{2}\right) < \frac{1}{8}$	(i)	$f\left(\frac{1}{3}\right) = \frac{1}{18}$	(P) $\int_0^1 f(x) dx > 1$
(II)	$f\left(\frac{1}{2}\right) > \frac{1}{8}$	(ii)	$f\left(\frac{1}{4}\right) = \frac{1}{8}$	(Q) $\int_0^1 f(x) dx > \frac{1}{6}$
(III)	$f\left(\frac{1}{3}\right) < \frac{1}{18}$	(iii)	$f\left(\frac{1}{6}\right) = 2\sin^2\left(\frac{1}{12}\right)$	(R) $\int_0^1 f(x) dx < \frac{1}{6}$
(IV)	$f\left(\frac{1}{4}\right) < \frac{1}{4}$	(iv)	$f\left(\frac{1}{8}\right) = 2\sin^2\left(\frac{1}{16}\right)$	(S) $\int_0^1 f(x) dx < \frac{1}{4}$

Question:

Which of the following options is the only CORRECT combination?

- A. (I) (iv) (Q)
- B. (II) (iii) (S)
- C. (II) (iv) (P)
- D. (IV) (iii) (S)